MOTION OF FLUID ELEMENT (KINEMATICS)

Before formulating the effects of forces on fluid motion (dynamics), first we consider the motion (kinematics) of a fluid in a flow field. When a fluid element moves in a flow field, it may under go **translation, linear deformation, rotation, and angular deformation** as a consequence of spatial variations in the velocity.

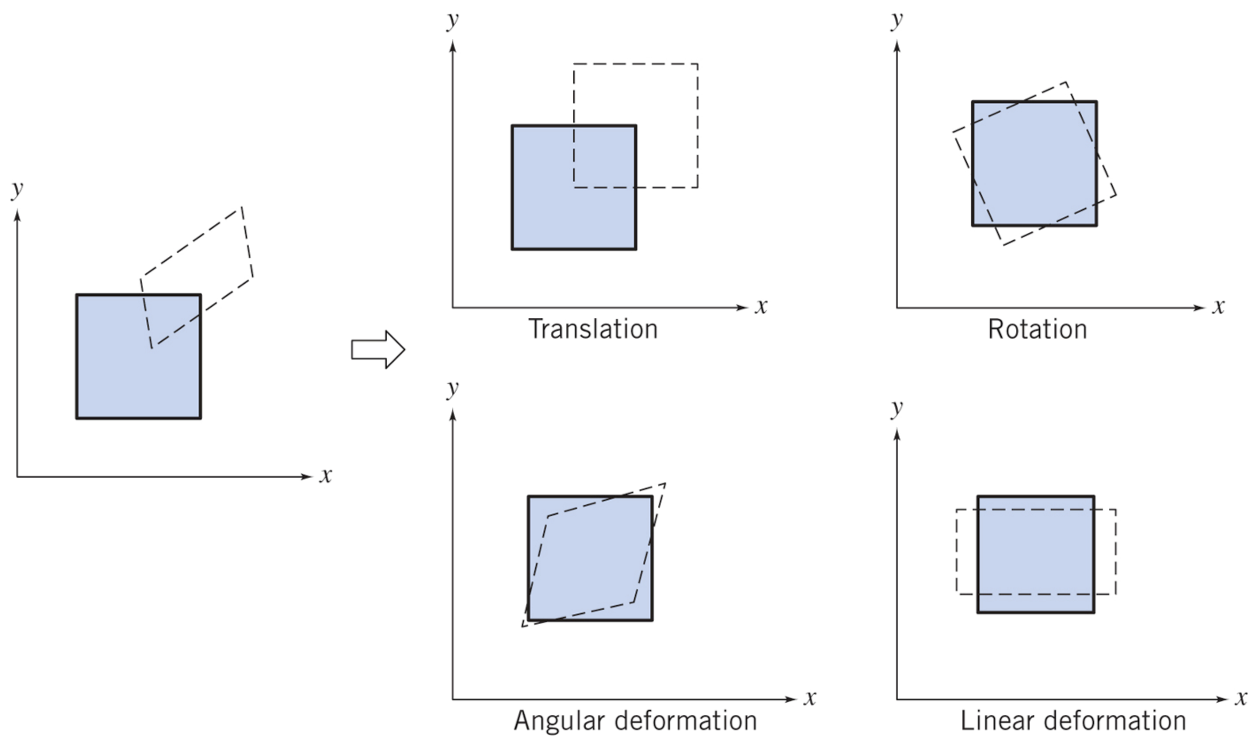


Figure. Pictorial representation of the components of fluid motion in a flow field.

Rate of Translation

Tranlation in unit time is equal to velocity,

$$\vec{V}_p(x,y,z,t) = u(x,y,z,t)\vec{i} + v(x,y,z,t)\vec{j} + w(x,y,z,t)\vec{k}$$

Acceleration of a Fluid Particle in a Velocity Field

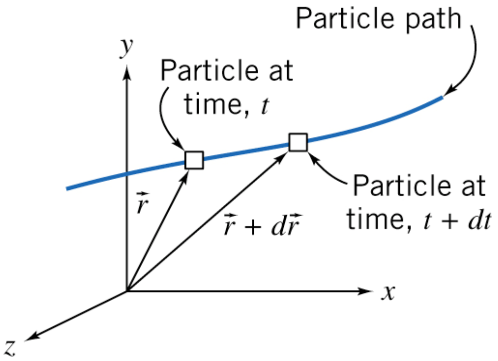


Figure. Motion of a particle in a flow field.

Consider a particle moving in a velocity field. At time *t*, the particle is at a position *x, y, z* and has a velocity $\left.\vec{V}_p\right]_t = \vec{V}(x,y,z,t)$

At time t+dt, the particle has moved to a new position, with coordinates *x+dx, y+dy, z+dz,* and has a velocity given by

$$\left.\vec{V}_p\right]_{t+dt} = \vec{V}(x+dx, y+dy, z+dz, t+dt)$$

The change in the velocity of the particle moving from location $\vec{r}$ to $\vec{r} + d\vec{r}$ is given by

$$d\vec{V}_p = \frac{\partial \vec{V}}{\partial x} dx_p + \frac{\partial \vec{V}}{\partial y} dy_p + \frac{\partial \vec{V}}{\partial z} dz_p + \frac{\partial \vec{V}}{\partial t} dt$$

Dividing both sides by **dt**, the total acceleration of the particle is obtained as

$$\vec{a}_p = \frac{d\vec{V}_p}{dt} = \frac{\partial \vec{V}}{\partial x}\frac{dx_p}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy_p}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz_p}{dt} + \frac{\partial \vec{V}}{\partial t}$$

Since $\quad \frac{dx_p}{dt} = u, \quad \frac{dy_p}{dt} = v \quad and \quad \frac{dz_p}{dt} = w$

then $\quad \vec{a}_p = \frac{d\vec{V}_p}{dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$

Acceleration of a fluid particle in a velocity field requires a special derivative, it is denoted by the symbol $\dfrac{D\vec{V}}{Dt}$ *or* $\dfrac{d\vec{V}}{dt}$

Thus, $$\frac{D\vec{V}}{Dt} = \vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

This derivation is called the **substantial**, the **material** or **particle derivative**.

The significance of the terms,

$$\vec{a}_p = \underbrace{\frac{D\vec{V}}{Dt}}_{\substack{\text{total}\\ \text{accelaration}\\ \text{of a particle}}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\ \text{acceleration}}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\substack{\text{local}\\ \text{acceleration}}}$$

The convective acceleration may be written as a single vector expression using the vector gradient operator, ∇.

Thus, $$u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = (\vec{V}\cdot\nabla)\vec{V}$$

$$\frac{D\vec{V}}{Dt} \equiv \vec{a}_p = (\vec{V}\cdot\nabla)\vec{V} + \frac{\partial \vec{V}}{\partial t}$$

It is possible to express the above equation in terms of three scalar equations as

$$a_{x_p} = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_{y_p} = \frac{Dv}{Dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_{z_p} = \frac{Dw}{Dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

The components of acceleration in cylindrical coordinates may be obtained by utilizing the appropriate expression for the vector operator ∇. Thus

$$a_{r_p} = V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z} + \frac{\partial V_r}{\partial t}$$

$$a_{\theta_p} = V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z} + \frac{\partial V_\theta}{\partial t}$$

$$a_{z_p} = V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z} + \frac{\partial V_z}{\partial t}$$

Example: The velocity field for a fluid flow is given by

$$\vec{V}(x,y,z,t) = x^2\vec{\imath} - 2xy\vec{j} + 3zt\vec{k}$$

Determine:

a) the acceleration vector,

b) the acceleration of the fluid particle at point P(1,2,3) and at time t = 1 sec.

To be completed in class

FLUID ROTATION

Definition: The rotation of a fluid particle is defined as the average angular velocity of any two mutually perpendicular line elements of particle in each corrdinate plane. Hence a particle may rotate about three coordinate axes. Thus, in general, rotation of a fluid element can be expressed as: $\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$

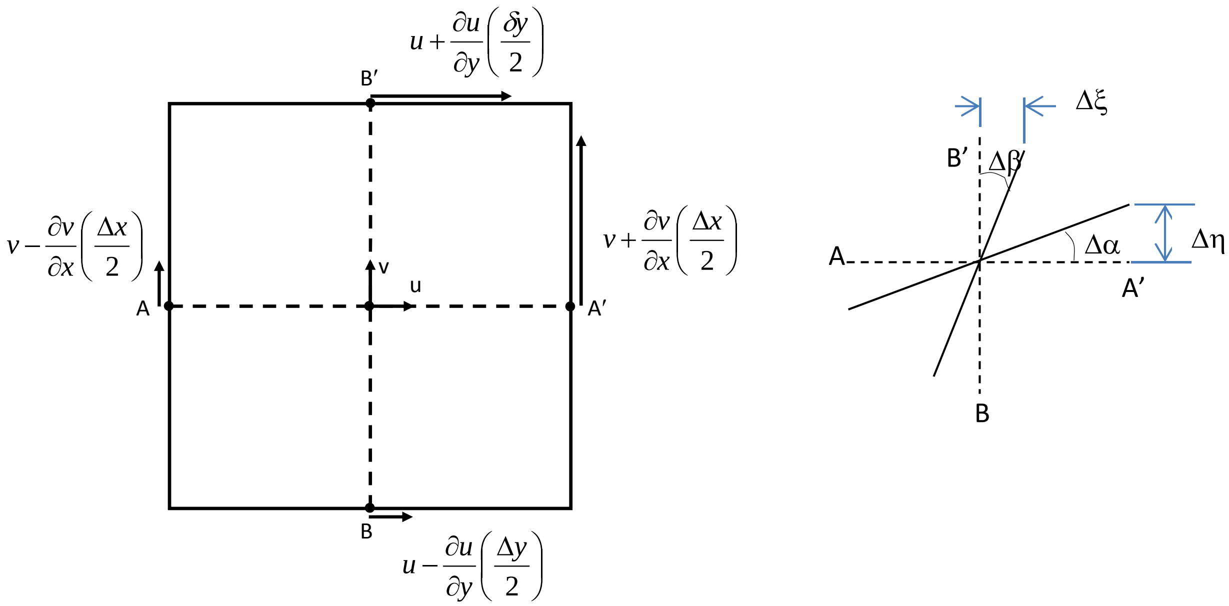


Figure. Rectangular fluid particle with two instantaneous perpendicular line AA’ and BB’.

By definition, the rotation of fluid element about z-axis can be written as

$$\omega_z = \frac{1}{2}\left(\omega_{AA'} - \omega_{BB'}\right) = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right)$$

$$\frac{d\alpha}{dt} = \lim_{\Delta t \to 0}\frac{\Delta\alpha}{\Delta t} = \lim_{\Delta t \to 0}\frac{\Delta\eta/(\Delta x/2)}{\Delta t} = \lim_{\Delta t \to 0}\frac{(\partial v/\partial x)\Delta x\,\Delta t/\Delta x}{\Delta t} = \frac{\partial v}{\partial x}$$

$$\Delta\eta = \left(v + \frac{\partial v}{\partial x}\frac{\Delta x}{2}\right)\Delta t - v\Delta t = \frac{\partial v}{\partial x}\frac{\Delta x}{2}\Delta t$$

$$\frac{d\beta}{dt} = \lim_{\Delta t \to 0}\frac{\Delta\beta}{\Delta t} = \lim_{\Delta t \to 0}\frac{\Delta\xi/(\Delta y/2)}{\Delta t} = \lim_{\Delta t \to 0}\frac{(\partial u/\partial y)\Delta y\,\Delta t/\Delta y}{\Delta t} = \frac{\partial u}{\partial y}$$

$$\Delta\xi = \left(u + \frac{\partial u}{\partial y}\frac{\Delta y}{2}\right)\Delta t - u\Delta t = \frac{\partial u}{\partial y}\frac{\Delta y}{2}\Delta t$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

By considering the rotation of pairs of perpendicular lines in the yz and xz planes, one can show that

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \quad \text{and} \quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

Then

$$\vec{\omega} = \omega_x \vec{\imath} + \omega_y \vec{j} + \omega_z \vec{k} = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{\imath} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}\right]$$

We recognize the term in the square brackets as

$$\text{curl}\vec{V} = \nabla \times \vec{V}$$

Then, in vector notation, we can write

$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{V}$$

The factor of ½ can be eliminated in above equation by defining a quantity called the ***vorticity***, $\vec{\zeta}$, to be twice the rotation,

$$\vec{\zeta} \equiv 2\vec{\omega} = \nabla \times \vec{V}$$

The vorticity is the measure of the rotation of a fluid element as it moves in the flow field.

In cylindrical coordinates the vorticity is

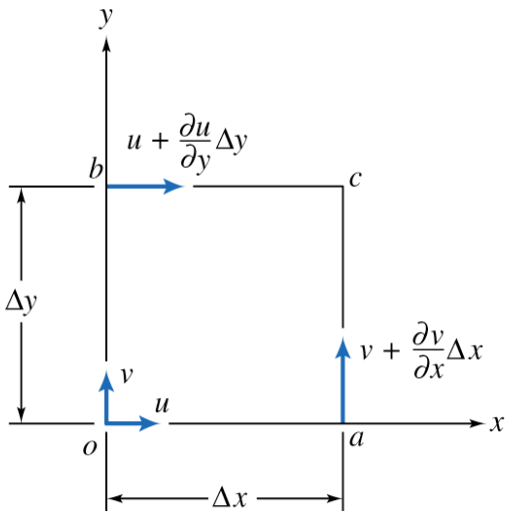
$$\nabla \times \vec{V} == \left(\frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right)\vec{e}_\theta + \left(\frac{1}{r}\frac{\partial rV_\theta}{\partial r} - \frac{\partial V_r}{\partial \theta}\right)\vec{e}_z$$

Circulation

The ***circulation***, Γ, is defined as the line integral of the tangential velocity component about a closed curve fixed in the flow,

$$\Gamma = \oint_C \vec{V} \cdot d\vec{s}$$

where, $d\vec{s}$ is an elemental vector, of the length ***ds***, tangent to curve; a **positive sense corresponds to a counterclockwise** path of integration around the curve.



For the closed curve ***Oacb,***

$$\Delta\Gamma = u\Delta x + \left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta y - \left(u + \frac{\partial u}{\partial y}\Delta y\right)\Delta x - v\Delta y$$

$$\Delta\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\Delta x \Delta y$$

$$\Delta\Gamma = 2\omega_z \Delta x \Delta y$$

$$\Gamma = \oint_C \vec{V} \cdot d\vec{s} = \int_A 2\omega_z dA = \int_A (\nabla \times \vec{V})_z dA$$

Thus, the circulation around a closed contour is equal to the total vorticity enclosed within it.

Example: Consider flow fields with purely tangential motion (circular streamlines): $V_r = 0$ and $V_\theta = f(r)$. Evaluate the rotation, vorticity, and circulation for rigid-body rotation, and "a forced vortex". Show that it is possible to choose f(r) so that the flow is irrotational; to produce "a free vortex".

To be completed in class

Angular Deformation of a Fluid Element

Definition: Angular deformation of a fluid element involves changes in the angle between two mutually perpendicular lines in the fluid.

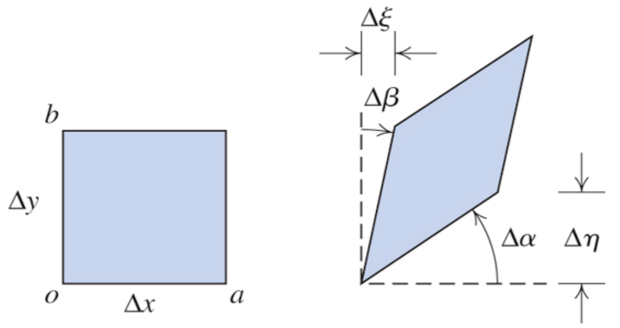


Figure. Angular deformation of a fluid element in a two dimensional flow field.

From the definition, the rate of angular deformation of fluid element can be expressed as

$$-\frac{d\gamma}{dt} = \frac{d\alpha}{dt} + \frac{d\beta}{dt}$$

$$\frac{d\alpha}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\alpha}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\eta / \Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{(\partial v / \partial x)\Delta x\, \Delta t / \Delta x}{\Delta t} = \frac{\partial v}{\partial x}$$

$$\frac{d\beta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\beta}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\xi / \Delta y}{\Delta t} = \lim_{\Delta t \to 0} \frac{(\partial u / \partial y)\Delta y\, \Delta t / \Delta y}{\Delta t} = \frac{\partial u}{\partial y}$$

Consequently, the rate of the angular deformation in the ***xy*** plane is obtained as

$$-\frac{d\gamma}{dt} = \varepsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

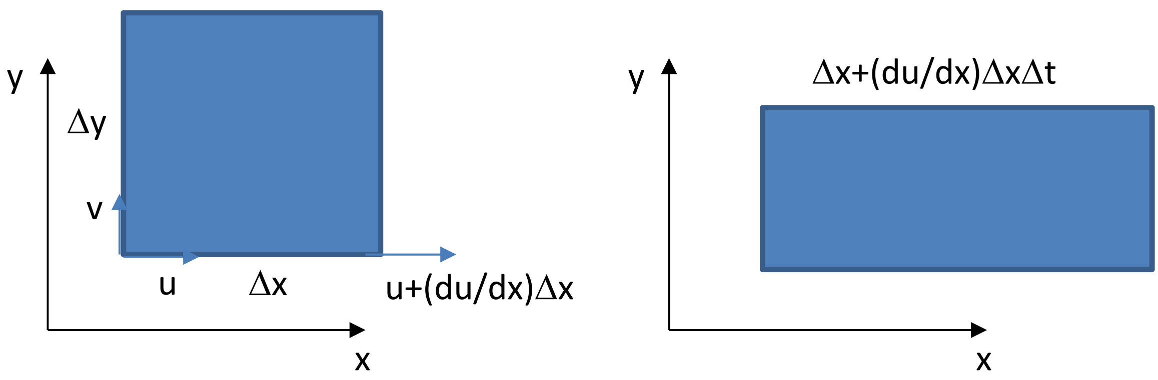
Rate of the angular deformation in the ***yz*** plane $\varepsilon_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$

Rate of the angular deformation in the ***zx*** plane $\varepsilon_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$

The shear stress is related to the rate of angular deformation through the fluid viscosity. For one-dimensional Newtonian laminar flow the shear stress is given by

$$\tau_{yx} = \mu \frac{\partial u}{\partial y}$$

Linear Deformation



Definition: Rate of linear deformation of a fluid element is defined as the change in the unit length in unit time in each coordinate direction.

Rate of linear deformation in x-dir: $\varepsilon_{xx} = \dfrac{[\Delta x + (\partial u / \partial x)\Delta x \Delta t] - \Delta x}{\Delta t} \Big/ \Delta x = \dfrac{\partial u}{\partial x}$

Similarly in y- and z-directions,

Rate of linear deformation in y-dir: $\varepsilon_{yy} = \dfrac{\partial v}{\partial y}$

Rate of linear deformation in z-dir: $\varepsilon_{zz} = \dfrac{\partial w}{\partial z}$

Change in the length of the sides of the fluid element may produce change in volume of the element. The rate of local instantaneous volume dilatation is given by

Volume dilation rate $= \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = \nabla \bullet \vec{V}$

MOMENTUM EQUATION

To derive the differential form of momentum equation, we shall apply **Newton's second law** to an infinitesimal fluid particle of mass ***dm***.

Newton's second law for **a finite system** is given by $\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{system}$

where the linear momentum, $\vec{P}$, of the system is given by

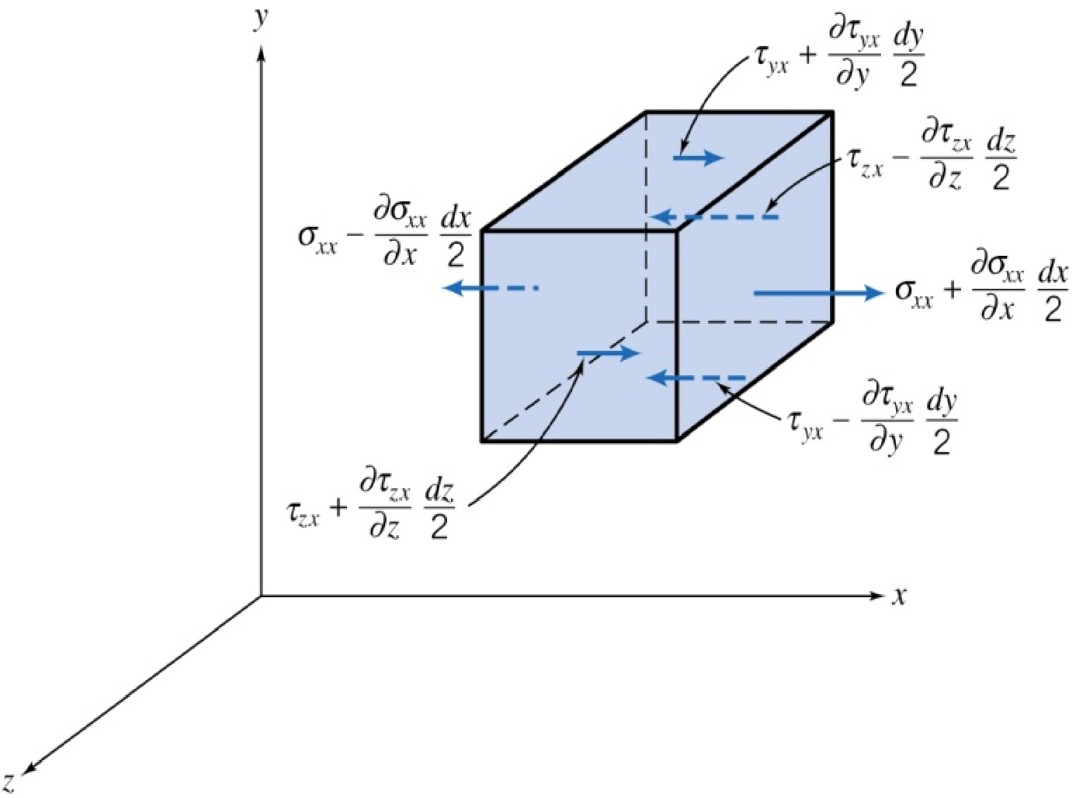
$$\vec{P}_{system} = \int_{mass\,(system)} \vec{V} dm$$

Then for **an infinitesimal** system of mass **dm,** Newton's second law is written as

$$d\vec{F} = dm \left.\frac{D\vec{V}}{Dt}\right)_{system} = dm \frac{D\vec{V}}{Dt}$$

$$\frac{D\vec{V}}{Dt} = \vec{a} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

$$\therefore \qquad d\vec{F} = dm\left[u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}\right]$$



Forces Acting on a Fluid Particle

The forces acting on a fluid element may be classified as **body forces** and **surface forces**. Surface forces include both normal forces and tangential (shear) forces. Surface force acting on a fluid element can be expressed in terms of stresses.

Stresses acting on a differential fluid element **in the x-direction** are shown in the figure.

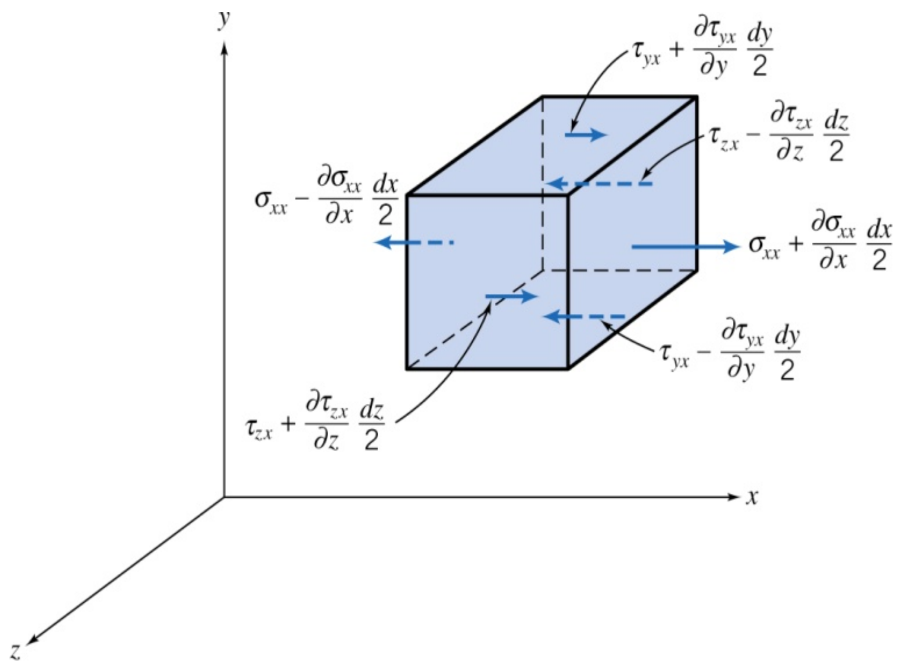


Figure. Stresses in the x direction on an element of fluid.

To obtain the net surface force in the *x*-direction, dF_{S_x}, we must sum the forces in the *x* direction.

$$dF_{S_x} = \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz - \left(\sigma_{xx} - \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz$$
$$+\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}\right)dxdz - \left(\tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}\right)dxdz$$
$$+\left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}\right)dxdy - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}\right)dxdy$$

By simplifying, we obtain

$$dF_{S_x} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dxdydz$$

When the gravity is only the body force acting, then the body force per unit mass in *x*-direction is given by

$$dF_{B_x} = g_x \rho dxdydz$$

Then the total net force in x-direction can b expressed as

$$dF_x = dF_{B_x} + dF_{S_x} = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dxdydz$$

One can derive similar expressions for the force components in the ***y* and *z* directions**.

$$dF_y = dF_{B_y} + dF_{S_y} = \left(\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) dxdydz$$

$$dF_z = dF_{B_z} + dF_{S_z} = \left(\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) dxdydz$$

Differential Momentum Equation

We have now formulated expressions for the components, dF_x , dF_y, and dF_z of the force, $d\vec{F}$, acting on the element of mass **dm**. If we substitute these expressions for the force components into *x, y*, and *z* components of equation, we obtain differential equations of motion.

$$\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right)$$

$$\rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right)$$

$$\rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right)$$

These three equations are the differential equations of motion for any fluid satisfying the **continuum assumption**. Before the equations can be used to solve problems, suitable expressions for the stresses must be obtained in terms of the velocity and pressure fields. Since the relation between stress and velocity is different for Newtonian and non-Newtonian fluids, to express the stresses in terms of velocity and pressure, we need to identify the type of fluid.

Newtonian Fluid: Navier-Stokes Equations

For a Newtonian fluid the viscous stress is proportional to the rate of shearing strain (angular deformation rate). The stresses may be expressed in terms of velocity gradients and fluid properties in rectangular coordinates as follows:

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\sigma_{xx} = -p - \frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial u}{\partial x}$$

$$\sigma_{yy} = -p - \frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial v}{\partial y}$$

$$\sigma_{zz} = -p - \frac{2}{3}\mu\nabla\cdot\vec{V} + 2\mu\frac{\partial w}{\partial z}$$

where ***p*** is the local thermodynamic pressure.

If these expressions are introduced into the differential equations of motion, we obtain

$$\rho\frac{Du}{Dt}=\rho g_x-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}\nabla\cdot\vec{V}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]$$

$$\rho\frac{Dv}{Dt}=\rho g_y-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}\nabla\cdot\vec{V}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]$$

$$\rho\frac{Dw}{Dt}=\rho g_z-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial w}{\partial z}-\frac{2}{3}\nabla\cdot\vec{V}\right)\right]$$

These equations of motion are called the Navier-Stokes equations. The equations are greatly simplified when applied to **incompressible flow with constant viscosity.** Under these conditions the equations reduce to

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

The Navier-Stokes equations in cylindrical coordinates, for constant density and viscosity, are given in the textbook.

For the case of **frictionless flow (μ = 0)** the equations of motion reduce to Euler's equation,

$$\rho\frac{D\vec{V}}{Dt}=\rho g-\nabla p$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}$$